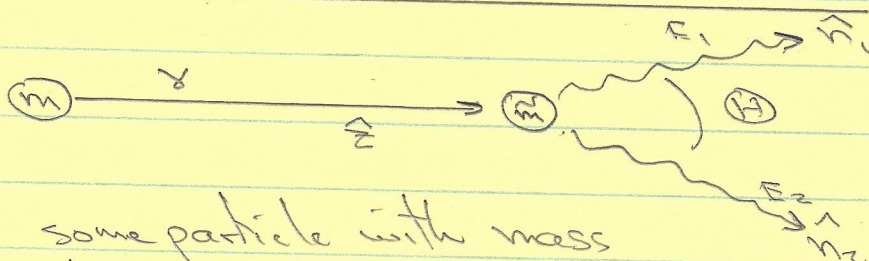


①

Positron annihilation revisited



m : some particle with mass

$\bar{m}$: its antiparticle

Question: What is the minimum scattering angle?

Need to find Θ ...

$$\text{CoE} \Rightarrow mc^2\gamma + \bar{m}c^2 = E_1 + E_2$$

$$\quad \quad \quad / mc^2 \Rightarrow$$

$$\gamma + 1 = \alpha_1 + \alpha_2 \quad (1)$$

note

$m \neq \bar{m}$

have identical

rest masses

(def'n of

antiparticle)

$$\text{CoM} \Rightarrow mc\vec{p}\gamma = \frac{E_1}{c}\hat{n}_1 + \frac{E_2}{c}\hat{n}_2$$

$$\quad \quad \quad / mc \Rightarrow$$

$$\vec{p}\gamma = \alpha_1\hat{n}_1 + \alpha_2\hat{n}_2 \quad (2)$$

(2)

①² - |②|² yields eq ③

$$(\gamma+1)^2 - \beta^2 \gamma^2 = (\alpha_1 + \alpha_2)^2 - |\alpha_1 \hat{n}_1 + \alpha_2 \hat{n}_2|^2$$

$$\underbrace{\gamma^2 + 2\gamma + 1 - \beta^2 \gamma^2}_{\gamma^2 - \beta^2 \gamma^2 = 1} = \cancel{\alpha_1^2 + 2\alpha_1 \alpha_2 + \alpha_2^2} - (\cancel{\alpha_1^2 + 2\alpha_1 \alpha_2 \hat{n}_1 \cdot \hat{n}_2 + \alpha_2^2})$$

$$2(\gamma+1) = 2\alpha_1 \alpha_2 (1 - \hat{n}_1 \cdot \hat{n}_2)$$

$$\frac{\gamma+1}{\alpha_1 \alpha_2} = 1 - \hat{n}_1 \cdot \hat{n}_2$$

$$\boxed{\hat{n}_1 \cdot \hat{n}_2 = 1 - \frac{\gamma+1}{\alpha_1 \alpha_2}} \quad (3)$$

From ① $\gamma+1 = \alpha_1 + \alpha_2$
eliminate α_2

$$\text{From ① } \alpha_2 = \gamma+1 - \alpha_1$$

∴

$$\boxed{\hat{n}_1 \cdot \hat{n}_2 = 1 - \frac{\gamma+1}{\alpha_1 (\gamma+1 - \alpha_1)}} \quad (4)$$

(3)

(4) is a relationship between $\hat{n}_1, \hat{n}_2 \nabla \alpha_1$
measuring one, determines the other

Let's see if there is an
extremum encoded in this
relationship!

From (4)

$$0 = \frac{d(\hat{n}_1, \hat{n}_2)}{d\alpha_1} = \frac{-(x+1)}{\alpha_1^2 (x+\alpha_1)^2} [x+1 - 2\alpha_1]$$

$$\Rightarrow \boxed{\alpha_1 = \frac{x+1}{2}} \text{ (5) is the extremum point of } \alpha_1$$

If $\alpha_1 = \frac{x+1}{2}$ at the extremum,

from (1) this implies

$$\boxed{\alpha_2 = \frac{x+1}{2}} \text{ (6) at the extremum}$$

as well!

$\therefore$ From (5) (6) $\stackrel{!}{=} (3)$ we determine
the extremum

(4)

$$\hat{n}_1 \cdot \hat{n}_2 \Big|_{\text{ext}} = 1 - \frac{(\gamma+1)}{\left(\frac{\gamma+1}{2}\right)\left(\frac{\gamma+1}{2}\right)}$$

$$= 1 - \frac{4}{\gamma+1}$$

$$\boxed{\hat{n}_1 \cdot \hat{n}_2 \Big|_{\text{ext}} = 1 - \frac{4}{\gamma+1}} \quad (7)$$

note that

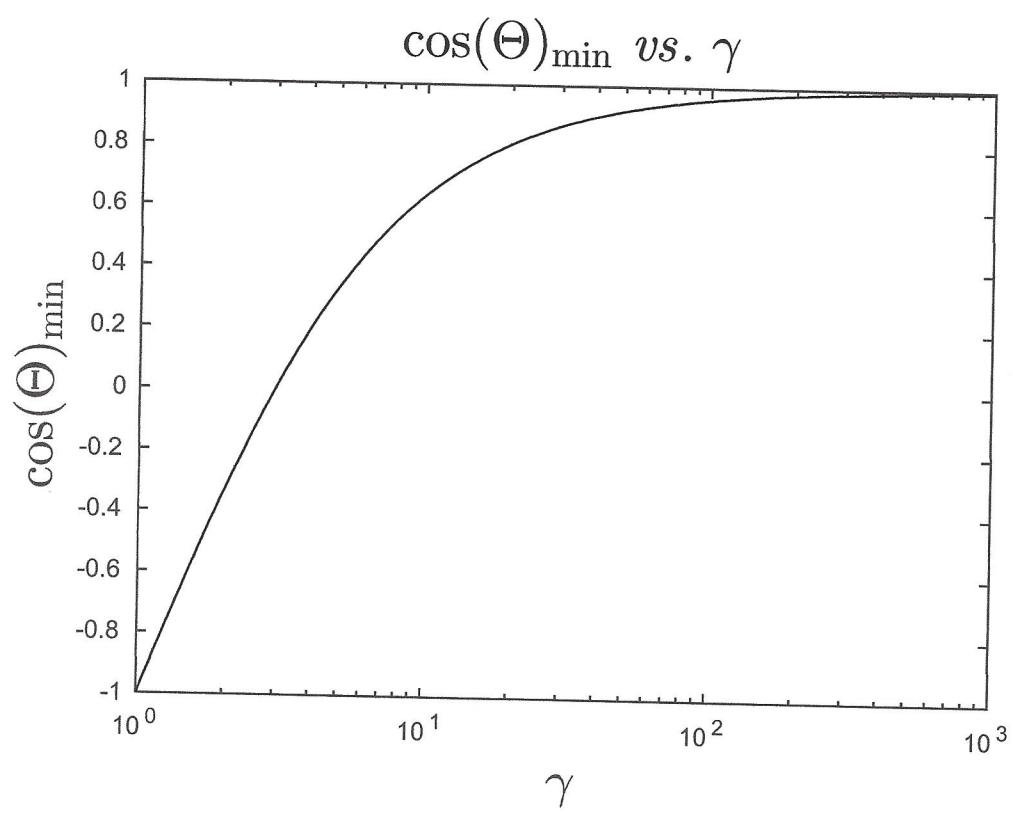
$$1 \leq \gamma < \infty$$

when $\gamma=1$ (at rest annihilation)
 $\hat{n}_1 \cdot \hat{n}_2 = -1$ ✓ (expected)
 (back to back γ s)

when $\gamma \rightarrow \infty$
 $\hat{n}_1 \cdot \hat{n}_2 = 1$ ✓✓ (expected)
 (everything just
 directed)

Anything else? See plot

Conclusion: There is a minimum opening angle
 reached when $E_1 = E_2$



(6)

Finally, let's return to ④ to discuss the kinematically accessible range of $\hat{n}_1 \cdot \hat{n}_2$

From ④

$$\frac{\gamma+1}{\alpha_1(\gamma+1-\alpha_1)} = 1 - \hat{n}_1 \cdot \hat{n}_2$$

$$(\gamma+1) = (1 - \hat{n}_1 \cdot \hat{n}_2) \alpha_1 (\gamma+1 - \alpha_1)$$

$$\text{or } \alpha_1^2 (1 - \hat{n}_1 \cdot \hat{n}_2) - \alpha_1 (\gamma+1) (1 - \hat{n}_1 \cdot \hat{n}_2) + (\gamma+1) = 0$$

This is a quadratic relation for α_1 ,

$$\textcircled{6} \quad \alpha_1 = \frac{(\gamma+1)(1 - \hat{n}_1 \cdot \hat{n}_2) \pm \sqrt{(\gamma+1)^2 (1 - \hat{n}_1 \cdot \hat{n}_2) - 4(\gamma+1)(1 - \hat{n}_1 \cdot \hat{n}_2)}}{2(1 - \hat{n}_1 \cdot \hat{n}_2)}$$

Note that the sum of the two roots is

$$\alpha_1^+ + \alpha_1^- = \gamma+1$$

So, the two roots give the two photon energies, from the CoE

So, in order for there to be 2 photons in the final state, the radical in (8) must be ≥ 0 .

$$\therefore (\gamma+1)(1-\hat{n}_1 \cdot \hat{n}_2)^2 - 4(\gamma+1)(1-\hat{n}_1 \cdot \hat{n}_2) \geq 0$$

So, if $\hat{n}_1 \cdot \hat{n}_2 \neq 0$

$$\boxed{(\gamma+1)(1-\hat{n}_1 \cdot \hat{n}_2) \geq 4} \quad (9)$$

If $\hat{n}_1 \cdot \hat{n}_2 = -1$

$$(\gamma+1)(2) \geq 4$$

$$\gamma+1 \geq 2$$

$$\gamma \geq 1$$

always true
always is this
solution

$$\gamma+1 \geq \frac{4}{1-\hat{n}_1 \cdot \hat{n}_2}$$

Let $\hat{n}_1 \cdot \hat{n}_2 = 1-\epsilon$ (almost a forward opening angle)

$$\Rightarrow \gamma+1 \geq \frac{4}{\epsilon} \Rightarrow \gamma \rightarrow \infty$$

(Already noted this)